Applications of Linear Models: Sections 1.3 – 1.5

We will do SOME (not all) of these problems in class.
The problems not done in class are intended as additional practice problems for exams and quizzes.
Some of these questions are from exams and quizzes from previous quarters.
Read sections 1.3, 1.4, 15 in the textbook. The textbook has additional worked examples and practice
homework and review problems in sections 1.3, 1.4, 1.5, 1.6

Finding The Equation Of A Line: Section 1.3

1. Find the equation of a line when given slope and y intercept
Strategy: Use slope intercept form of a line

a. A long distance telephone carriers (for landline phones) has a monthly charge of $20 plus charges of
$.05 per minute for calls. x = minutes of phone calls, y = cost

b. A car rental company charges $243 for a one-way weekend rental from NY to Boston, with an
additional charge of 19 cents per mile. x = miles driven ; y = total cost

c. The value of a piece of manufacturing equipment is $66000 and decreases at the rate of $3000 per year.
x = age of equipment; y = value

2. Find the equation of a line when given slope and one point on the line
Strategy: Use point slope form of a line

a. It costs $10 to ship 3 books from an online bookstore; for every additional book the total shipping
cost increases by $2. x = number of books; y = shipping cost.

b. It costs Sugar & Spice Bakery $200 to make 120 cupcakes; for every additional 12 cupcakes, it costs $9.
x = number of cupcakes y = total cost

c. PrettyPix Inc sells picture frames. Consumers will buy 200 picture frames if the price is $15 but will buy 60
fewer frames for every $5 increase in price. x = price, y = quantity (number of frames)

d. Ike's Ice Cream Palace sells 30 gallons of ice cream if the temperature outside is 50 degrees F.
They sell 1 additional gallon for every two degree increase in temperature.
x = temperature in degrees Fahrenheit; y = gallons of ice cream.

e. City ClockWorks sells decorated souvenir wall clocks. They sell 1000 clocks if the price is $24. For every one
dollar decrease in price, they sell 45 additional clocks.   x = price, y = quantity (number of clocks)

3. Find the equation of a line when given two points on the line
Strategy: Use both points to find the slope of the line
Then use the slope and either of the points to write the point-slope form of a line

a. Keisha makes jewelry and sells it online on Etsy. It costs her $350 to produce 10 bracelets and it
costs her $950 to produce 40 bracelets.   x = number of bracelets ; y = cost

b. A sports team wants to sell tee shirts for a special event. A tee shirt manufacturer says it would cost $20000 to
produce 1000 of the tee shirts and it would cost $50000 to produce 3000 tee shirts.
x = number of tee shirts; y = cost

c. The value of a piece of machinery is $50000 at the end of 5 years and $35000 at the end of 8 years.
x = age of equipment ; y = value of equipment

Cost Functions: y = C(x)
      The total cost of producing an item typically consists of two parts
          the fixed cost that does not depend on the number of items produced
          the variable cost depends on the additional cost to produce an extra item

Total Cost C(x) = Fixed Cost + Variable Cost
Total Cost C(x) = Fixed Cost + (Marginal Cost)(number of items produced)

The y intercept is the_____

The slope is the _____

Discussion: What might be some types of things included in fixed cost and in variable cost?

Revenue Functions: y = R(x)
      Revenue means income; Revenue = price per item multiplied by the number of items sold
Revenue R(x) = (price)(quantity)
      The best way to remember the "formula" for revenue is as the "word equation" above,
      because sometimes x is price but other times x is quantity.

Revenue is not the same as profit. Revenue is money coming in. Cost is money going out.
Profit is money coming in MINUS money going out.

Profit = Revenue – Cost ; P(x) = R(x) – C(x)
      When Revenue = Cost, this is a Break Even point and profit = 0
      If Revenue > Cost, there is a profit
      If Revenue < Cost, there is a loss (the profit function is negative)

4 At Tony's Pizza Palace the fixed cost of making pizzas for one day is $300. The variable cost to make a pizza is $5 per pizza. x = number of pizzas ; y = cost
    a. Write the cost function.
    b. If Tony expects to sell 60 pizzas on a typical day, what should he charge for a pizza in order to at least cover his costs?
    c. If Tony sells pizzas for $15 each, how many pizzas does he need to sell in order to break even?
    d. What is the profit or loss if Tony sells 20 pizzas for $15 each? 50 pizzas for $15 each?

5. A factory makes protective cell phone cases. It costs $2000 to produce 100 cell phone cases; 500 cases cost $5000 to produce. Suppose the cell phone cases sell for $10 each. x = number of phone cases; y is in dollars
    a. Find the fixed and variable costs and write the cost function for producing cell phone cases.
    b. Find the revenue function and find the profit function.
    c. How many items must be sold to break even? What is the cost and revenue at that point??

6. Keisha makes jewelry and sells it online on Etsy. It costs her $350 to produce 10 bracelets and it costs her $950 to produce 40 bracelets. Keisha sells her bracelets for $27.50 each. Find the cost and revenue functions and use them to determine how many bracelets she needs to sell in order to break even.
x = number of bracelets; y is in dollars

7. A company producing low flow shower heads that conserve water has a fixed cost of $4000 and a variable cost of $7 per shower head. The shower heads sell at a price of $12 each.
x = number of shower heads; y represents cost or revenue in dollars.
    a. Find the cost, revenue and profit functions.
    b. What is the profit or loss when 600 shower heads are produced and sold?
    c. What is the profit or loss when 1200 shower heads are produced and sold?
    d. Find the break even point (both number of shower heads and the cost and revenue).

Supply and Demand; Equilibrium: Sections 1.4 and 1.5

S(x) = supply function
S(x)= amount of product that suppliers are willing to produce when the price is x dollars

When price increases, supply _____; slope is _____

D(x) = demand function
D(x) = amount of product that consumers are willing to buy when the price is x dollars.

When price increases, demand _____; slope is _____.

Equilibrium occurs when Supply = Demand
      1) Set supply function = demand function
      2) Solve for x to find price when supply = demand
      3) Substitute equilibrium price into supply function or demand function to find the amount of the
         product that is produced by suppliers and bought by consumers at the equilibrium point

8. PrettyPix produces decorative picture frames. If the price is $15, they are willing to produce 125 frames. If the price is $22, they are willing to produce 216 frames. Consumers will buy 200 picture frames if the price is $15 but will buy 60 fewer frames for every $5 increase in price. x = price, y = quantity
a. Find the supply and demand functions
b. Find the equilibrium point.
c Graph the supply and demand functions and label the (x,y) coordinates of the equilibrium point.
d. If the price is $16, does supply exceed demand or is supply less than demand? By how much?
e If the price is $21, does supply exceed demand or is supply less than demand? By how much?

9. A company makes and sells puzzles for children. x = price ; y = number of puzzles
For wooden puzzles: the supply function is y = S(x) = 80x−900 and the demand function is y = D(X) = 1200−70x
a. How many wooden puzzles are they able to make when the price is $15?
   How many wooden puzzles are consumers willing to buy when the price is $15?
   Does supply exceed demand or does demand exceed supply?
b. How many wooden puzzles are they able to make when the price is $12?
   How many wooden puzzles are consumers willing to buy when the price is $12?
   Does supply exceed demand or does demand exceed supply?
c. Find the equilibrium point
d. Graph the supply and demand functions and label the (x,y) coordinates of the equilibrium point.
e. For every $1 increase in price, what happens to the supply of wooden puzzles?
f. For every $1 increase in price, what happens to the demand for wooden puzzles?

10. A company makes and sells puzzles for children. x = price ; y = number of puzzles
For cardboard puzzles: the supply function is y = S(x) = 60x−120 and the demand function is y = D(X) = 480−40x
a. How many cardboard puzzles are they willing to make when the price is $4?   How many cardboard puzzles are consumers willing to buy when the price is $4? Does supply exceed demand or does demand exceed supply?
b. How many cardboard puzzles are they able to make when the price is $7?   How many cardboard puzzles are consumers willing to buy when the price is $7? Does supply exceed demand or does demand exceed supply?
c. Find the equilibrium point
d Graph the supply and demand functions and label the (x,y) coordinates of the equilibrium point.
e. For every $1 increase in price, what happens to the supply of cardboard puzzles?
f. For every $1 increase in price, what happens to the demand for cardboard puzzles?

APPLICATIONS: LINEAR MODELS: SYSTEMS OF EQUATIONS
Using Systems of Linear Equations to compare costs or income:

11. *Problem 7 is adapted from OpenStax College Algebra available for download free at*
    https://openstaxcollege.org/files/textbook_version/low_res_pdf/49/EditedCollegeAlgebra-2015-06-05-LR.pdf

Two different long distance telephone carriers (for landline phones) offer the following plans that a person is considering. Company A has a monthly charge of $20 plus charges of $.05 per minute for calls. Company B has a monthly fee of $5 and charges $.10 per minute for calls. x = number of minutes of class, y = cost

   a. Find the linear model for the total cost of Company A's plan, using x as the number of minutes of calls per month

   b. Find the linear model for the total cost of Company B's plan, using x as the number of minutes of calls per month

   c. For how many minutes do both plans produce the same cost?

   d. On a separate piece of graph paper, graph each function.
      Use $0 \le x \le 500$ minutes where each box represents 50 minutes
      and $0 \le y \le$ where each box represents $10.

   f. For what interval(s) of minutes does Company B's plan have the lowest cost?

12. Thuy is comparing the membership costs at two gyms.

   Gym F: $200 annual fixed cost plus $25 per month

   Gym G. $50 per month

   x = number of months of membership
   y = cost

   a. Write the annual cost functions f, g, for membership for each gym as a function of the number of months of membership

   b. On a separate piece of graph paper, graph each function for $0 \le x \le 12$ months; $0 \le y \le 600$
      Scale the x axis in months and the y axis so that each box is $50.

   c. In 2 to 4 complete sentences, compare the costs, analyzing which is gym least expensive over what periods of time.

13. Assaf is considering 3 different sales jobs selling networking systems software

   At FlashNet (F), he has been offered monthly wages consisting of a base salary of $2500 per month, plus a commission of 6% of his monthly sales

   At Galaxy Network Solutions (G), he has been offered compensation based only on commission of 10% of his monthly sales with no base salary.

   At High Speed Networking (H) his salary would be $4000 per month, with no commission.

   a. Write the functions f, g, h for y = wages at each company based on x = amount of monthly sales

   b. Algebraically find the points of intersection of each pair of functions.
      For what values of (x,y) do f and g intersect?
      For what values of (x,y) do f and h intersect?
      For what values of (x,y) do g and h intersect?

   c. Graph each function.

   d. Use a highlighter to show on the graph which company offers the highest wages at what levels of sales.

   e. Write 3 to 5 complete sentences analyzing which company offers the highest wages at what level of sales

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