1. (25 points) At what initial speed V_0 must the basketball player throw the ball, at angle θ_0 above the horizontal, to make the foul shot? (in terms of d_1, d_2, h_1, h_2, θ_0, and constant)
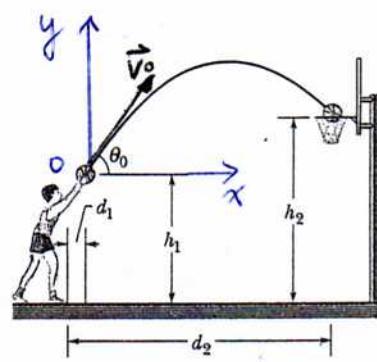
projectile. choose x. y. origin

$$t = 0, \quad \vec{V}_0 = V_0 \cos\theta_0 \, \hat{i} + V_0 \sin\theta_0 \, \hat{j}$$

$$X_0 = y_0 = 0$$

at t, make the foul shot

$$x = x_0 + V_{0x} t + \frac{1}{2} a_x \cdot t^2$$

$$d_2 - d_1 = V_0 \cos\theta_0 \cdot t \quad\text{———①}$$

$$y = y_0 + V_{0y} t - \frac{1}{2} g t^2$$

$$h_2 - h_1 = V_0 \sin\theta_0 \, t - \frac{1}{2} g t^2 \quad\text{———②}$$

from ① $\quad t = \dfrac{d_2 - d_1}{V_0 \cos\theta_0}$ into ②

$$h_2 - h_1 = V_0 \sin\theta_0 \cdot \frac{d_2 - d_1}{V_0 \cos\theta_0} - \frac{1}{2} g \left(\frac{d_2 - d_1}{V_0 \cos\theta_0} \right)^2$$

$$h_2 - h_1 = \tan\theta_0 (d_2 - d_1) - \frac{1}{2} g \left(\frac{d_2 - d_1}{V_0 \cos\theta_0} \right)^2$$

$$\frac{1}{2} g \cdot \left(\frac{d_2 - d_1}{V_0 \cos\theta_0} \right)^2 = \tan\theta_0 (d_2 - d_1) - (h_2 - h_1)$$

$$V_0^2 = \frac{g (d_2 - d_1)^2}{2 \cdot \cos^2\theta_0 \left(\tan\theta_0 (d_2 - d_1) - (h_2 - h_1) \right)}$$

$$V_0 = \frac{(d_2 - d_1)}{\cos\theta_0} \sqrt{\frac{g}{2 [\tan\theta_0 (d_2 - d_1) - (h_2 - h_1)]}}$$

2. (25 points) Car A is traveling north at 20 m/s crosses the intersection, at the instant car B starts from rest 50m east of the intersection and moves west with a constant acceleration of 2 m/s2. t seconds after A crosses the intersection, find (a) the position of B relative to A, (b) the velocity of B relative to A, (c) the acceleration of B relative to A.

$$2-D \text{ Motion}$$

$$\text{Position} \quad \vec{r_A} = x_A \hat{i} + y_A \hat{j} = 0\hat{i} + y_A$$

$$y_A = \cancel{y_{oA}}^0 + V_{oA}t + \tfrac{1}{2}\cancel{a_A}^0 t^2 = Vt$$

$$\vec{r_A} = 20t \,\hat{j}$$

$$\vec{r_B} = x_B \hat{i} + y_B \hat{j} = x_B \hat{i} + 0$$

$$x_B = x_{oB} + \cancel{V_{oB}t}^0 + \tfrac{1}{2}a_B t^2$$

$$= 50 - \tfrac{1}{2} \times 2 \cdot t^2$$

$$\vec{r_B} = (50 - t^2)\,\hat{i}$$

(a) $\quad \vec{r_B} - \vec{r_A} = (50 - t^2)\hat{i} - 20t\,\hat{j}$

(b) $\quad \vec{V_B} = \dfrac{d\vec{r_B}}{dt} = -2t\,\hat{i}$

$\quad \vec{V_A} = \dfrac{d\vec{r_A}}{dt} = -20\,\hat{j}$

$\quad \vec{V_B} - \vec{V_A} = -2t\,\hat{i} - 20\,\hat{j}$

(c) $\quad \vec{a_B} = \dfrac{d\vec{V_B}}{dt} = -2\,\hat{i}$

$\quad \vec{a_A} = \dfrac{d\vec{V_A}}{dt} = 0$

$\quad \vec{a_B} - \vec{a_A} = -2\,\hat{i}$
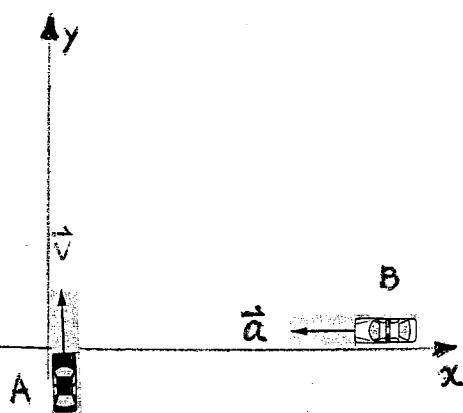
3. (25 points) A small object of mass m is suspended from a string attached to a fixed point on the ceiling of a car. When the car moves with acceleration a along an inclined plane of angle β, the object will deflect and the string will make an angle θ with the vertical. Find angle θ (in terms of β and a).

free body diagram

two forces $\vec{T}$, $m\vec{g}$ on the small object

choose x, y axis

apply Newton's second Law
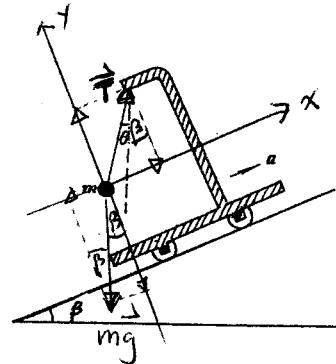
x: $T \cdot \sin(\theta + \beta) - mg \cdot \sin\beta = ma$

$T \sin(\theta + \beta) = mg \sin\beta + ma$ ———①

y: $T \cos(\theta + \beta) - mg \cos\beta = 0$

$T \cdot \cos(\theta + \beta) = mg \cdot \cos\beta$ ———②

①/② to eliminate T to solve θ

$\tan(\theta + \beta) = \dfrac{mg \sin\beta + ma}{mg \cdot \cos\beta}$

$\tan(\theta + \beta) = \dfrac{g \sin\beta + a}{g \cdot \cos\beta}$

$\theta + \beta = \tan^{-1}\left(\dfrac{g \sin\beta + a}{g \cos\beta}\right)$

$\therefore \theta = \tan^{-1}\left(\dfrac{g \sin\beta + a}{g \cos\beta}\right) - \beta$

4. (25 points) A small block with mass m is placed inside an inverted cone that is rotating about a vertical axis such that the time for one revolution of the cone is T. The walls of the cone make an angle β with the vertical. The coefficient of static friction between the block and the cone is μs. If the block is to remain stationary relative to the rotating cone at a constant height h, what is the maximum value of T? (in terms of h, β, μs and constant)

Circular motion of radius $r = h \cdot \tan\beta$
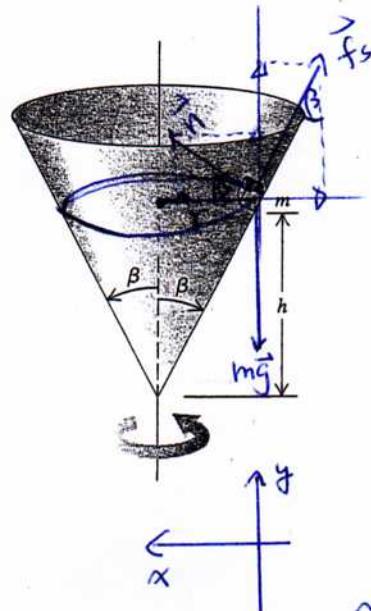
max T. $T = \dfrac{2\pi r}{V}$ min V

block has a tendency to slide down.

So static friction on the block up $\nearrow$

Three forces $\vec{n}$, $\vec{f_s}$, $\vec{mg}$

choose x, y.

apply Newton's second law

x: $n \cdot \cos\beta - f_s \cdot \sin\beta = ma_c = m\dfrac{v^2}{r}$

y: $n \cdot \sin\beta + f_s \cdot \cos\beta - mg = 0$

$$f_{s\,max} = \mu_s \cdot n$$

$$n \cdot \cos\beta - \mu_s \cdot n \sin\beta = m \cdot \dfrac{v^2}{r} \quad\text{——①}$$

$$n \sin\beta + \mu_s n \cos\beta = mg \quad\text{——②}$$

①/②

$$\dfrac{\cos\beta - \mu_s \sin\beta}{\sin\beta + \mu_s \cos\beta} = \dfrac{v^2}{gr} = \dfrac{\left(\dfrac{2\pi r}{T_{max}}\right)^2}{gr} = \dfrac{4\pi^2 h \cdot \tan}{g T_{max}^2}$$

$$T_{max}^2 = \dfrac{4\pi^2 h \cdot \tan\beta}{g} \cdot \dfrac{\sin\beta + \mu_s \cdot \cos\beta}{\cos\beta - \mu_s \sin\beta}$$

$$T_{max} = 2\pi \sqrt{\dfrac{h \cdot \tan\beta}{g} \cdot \dfrac{\sin\beta + \mu_s \cdot \cos\beta}{\cos\beta - \mu_s \cdot \sin\beta}}$$