CENTER OF MASS

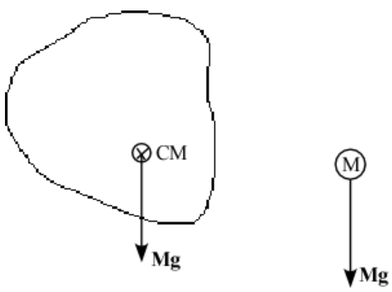
The motion of a system may appear to be quite difficult to describe because different particles making up the system will have different position, velocity, and acceleration. However, as we will see, it is not difficult to describe the motion of a system if we consider the motion of a special point called the center of mass. Two questions immediately arise: (1) What do we mean by the center of mass? and (2) How do we find the center of mass for a system?. The answer to the first question is given below. We will prove it later by applying N2L to the system

The center of mass of a body or a system of particles is:

a) the point that moves as though all the mass was concentrated at that point and
b) the resultant external force was applied at that point.

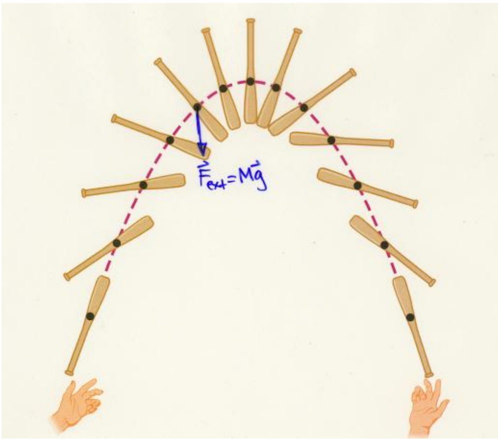
That is, the system or body moves as if the resultant external force was applied to a single particle of mass M located at the center of mass. We can also think of center of mass as the average position of the system's mass or a weighted-mass average of the system's mass.

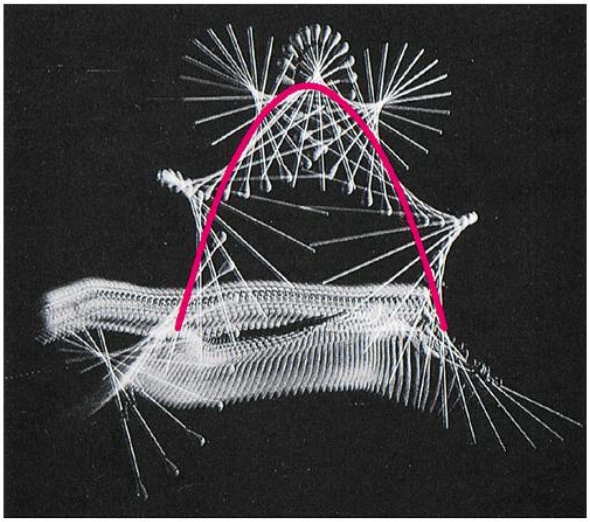
Ex. Object in Free-Fall



The object moves in Free-Fall as if all the mass was concentrated on a single particle of mass M located at the center of mass and the net external force Mg was acting at the center of mass.

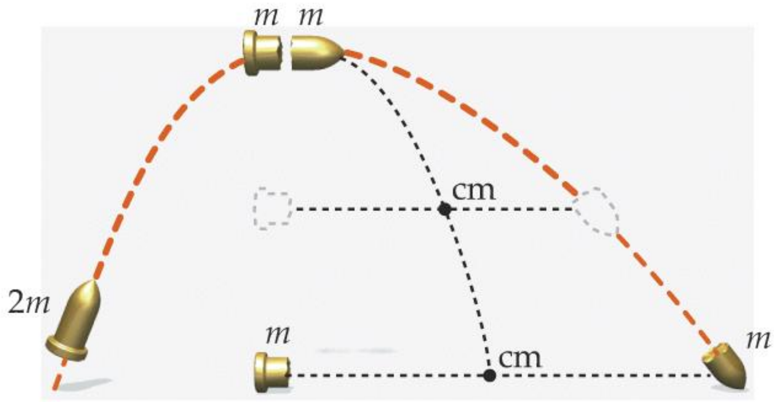
Ex. Thrown Baseball Bat/Baton





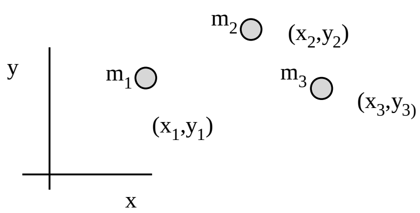
Once the bat is thrown, the center of mass moves in a simple parabolic path - just like a single particle of mass M located at the center of mass and the net external force Mg acting at that point.

Ex. Launched projectile the breaks up



Center of Mass for a System of Particles

Consider a system of 3 particles as shown below:



The x and y-coordinates of the center of mass are given by:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

For an n-particle system:

$$x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{M}, \quad y_{cm} = \frac{\sum_{i=1}^{n} m_i y_i}{M}, \quad z_{cm} = \frac{\sum_{i=1}^{n} m_i z_i}{M}$$

The center of mass can also be located by its position vector:

$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

$$\vec{r}_{cm} = \frac{\sum m_i x_i \hat{i} + \sum m_i y_i \hat{j} + \sum m_i z_i \hat{k}}{M}$$

$$\vec{r}_{cm} = \frac{\sum m_i \quad x_i \hat{i} + y_i \hat{j} + z_i \hat{k}}{M}$$

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{M}$$ (Center of Mass for a System of Particles)

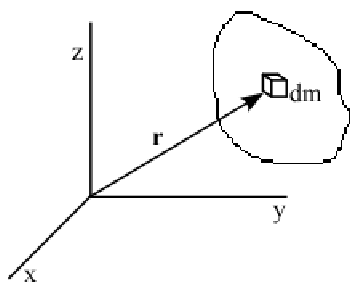
Where $\vec{r}_i = x_i \hat{i} + y_i \hat{j} + z_i \hat{k}$ is the position of the i[th] particle.

Center of Mass for an Extended Body

To find the CM for an extended body we divide the body into a large number of mass elements Δm_i and take the limit of $\vec{r}_{cm}$ as $\Delta m_i \to 0$:

$$\vec{r}_{cm} = \frac{1}{M} \lim_{\Delta m_i \to 0} \sum_{i=1}^{\infty} \Delta m_i \vec{r}_i$$

$$\boxed{\vec{r}_{cm} = \frac{1}{M} \int \vec{r}\, dm}$$



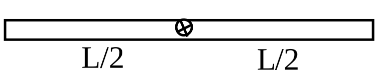
In component form:

$$\boxed{x_{cm} = \frac{1}{M} \int x\, dm \quad y_{cm} = \frac{1}{M} \int y\, dm \quad z_{cm} = \frac{1}{M} \int z\, dm}$$

For a homogeneous, symmetric body the center of mass is at its geometric center.

Ex.

Uniform Rod



Uniform Sphere